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Συντονιστές: Β. Γρηγοριάδης, Κ. Τσαπρούνης

Ομιλητές

Thilo Weinert (Independent Researcher)

On Untranscendable Linear Orders

Set theory has its roots in logic but in analysis as well. Georg Cantor investigated criteria for the convergence of trigonometric series and this lead him to analyse infinite suborders of the real line.

Afterwards, the investigation of abstract linear orders became part of set theory but is arguably less prominent than that of the closely related notions of cardinal and ordinal numbers. Often times notions can be generalised from the class of ordinals to the class of all linear orders, for example the operations of addition and multiplication. We are going to explore *untranscendability* which generalises the notion of being a δ-number. An order-type ρ is *transcendable* if it can be conceived of as a suborder of a product of two order-types smaller than ρ and *untranscendable* otherwise. Thus transcendability is a multiplicative analogue of decomposability—an order type τ is *decomposable* if it is the sum of two order-types smaller than τ and *indecomposable* otherwise. An order type φ is *strongly indecomposable* if for an order of type φ on a set X and any $Y \subset X$ the order X can be order-preservingly embedded into at least one of the induced suborders on Y and $X \setminus Y$, respectively.

We are going to show that all but one of the untranscendable linear orders are indecomposable, 2 being the only exception. Things are getting interesting when indecomposability is replaced by strong indecomposability—a natural example of an untranscendable order type is the one of the reals, λ. But λ is not necessarily strongly indecomposable, in fact, according to the axiom of choice, it is not. Yet the above result can be salvaged for class of σ-scattered linear orders—recall that an order is *scattered* if it does not contain a the order type of the rationals, η, and that it is σ-scattered if it can be decomposed into countably many scattered orders. This is to say that 2 is the unique untranscendable σ-scattered order type failing to be strongly indecomposable. This can also be proved for larger classes of linear orders assuming the proper forcing axiom.

We are going to discuss the notions mentioned above, give proof ideas or, time permitting, actual proofs, and close mentioning a few open questions.

This is joint work with Garrett Ervin and Alberto Marcone.

Αριστοτέλης Παναγιωτόπουλος (Kurt Gödel Research Center, University of Vienna)

Incompleteness theorems for observables in general relativity

Formulating a theory of quantum gravity is a major open problem in mathematical physics. Some of the core technical and epistemological difficulties come from the fact that General Relativity (GR) is 'generally covariant', i.e. invariant under change of coordinates by the arbitrary diffeomorphism of the ambient manifold. The Problem of Observables is a famous instance of the difficulties that general covariance brings into quantization: no non-trivial diffeomorphism-invariant quantity has ever been reported on the collection of all spacetimes. It turns out that there is a good reason for this. In this talk, I will present my joint work with Marios Christodoulou and George Sparling, where we employ methods from Descriptive Set Theory in order to show that, even in the space of all vacuum solutions, no complete observables can be Borel definable. That is, the problem of observables is to 'analysis' what the Delian problem is to 'straightedge and compass'.

Ιωάννης Σουλδάτος (Αριστοτέλειο Πανεπιστήμιο Θεσσαλονίκης)

A diagonalization principle for functions from ω_1 to ω_1

We will introduce a diagonalization principle for functions from ω_1 to ω_1 and prove that it is independent from the Axioms of ZFC. Then we will apply this independence result to a construction of Hjorth about characterizing cardinals.

The first 5-10 minutes of the talk will be accessible to Mathematicians with no or little background in Logic and independence proofs.

This is joint work with Philipp Lücke.