19ο Πανελλήνιο Συνέδριο Μαθηματικής Ανάλυσης Δ υναμικά Συστήματα – Μαθηματική Μοντελοποίηση $18\text{-}20~\Delta$ εκεμβρίου 2025

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Ομιλητές

Σταύρος Αναστασίου (Πανεπιστήμιο Δυτικής Μακεδονίας)

Planar vector fields in the kernel of a 1-form

We classify, up to a natural equivalence relation, vector fields of the plane which belong to the kernel of a 1–form. This form can be closed, in which case the vector fields are integrable, or not, in which case the differential of the form defines a, possibly singular, symplectic form. In every case, we provide a fairly complete list of local models for such fields and construct their transversal unfoldings. Thus, the local bifurcations of vector fields of interest can be studied, among them being the integrable fields of the plane.

Παναγιώτης Βασιλόπουλος (Εθνικό Μετσόβιο Πολυτεχνείο)

Invariant Manifolds in the Second-Order Maxwell-Bloch Equations

We consider the dynamics of the general Maxwell–Bloch equations modified by a linear restoring term. We use the slowly varying amplitude approximation to eliminate second-order derivatives in the polarization, while retaining the second-order derivatives in the electric field. This model governs a two-level laser in a ring cavity. For a certain class of lasers, this system exhibits two separated time scales and forms a singularly perturbed, hyperbolic system with three distinct characteristics. We extend the infinite-dimensional geometric singular perturbation theory of Menon and Haller [G. Menon and G. Haller, SIAM J. Math. Anal., 33(2), 315-346, 2001] to the general Maxwell–Bloch equations by proving the persistence of an arbitrarily smooth, infinite-dimensional slow manifold under an unbounded perturbation. The attractor of the system is contained in this manifold. We use a modified graph transform method to prove the existence of this manifold, together with a cone condition that defines the graph transform. Finally, observing that the first- and second-order formulations exhibit the same asymptotic behavior, we infer that the lack of a slowly varying amplitude approximation for the electric field is effectively compensated by the damping term, thereby providing an a posteriori justification for the approximation.

Joint work with V. Rothos and A. Charalampopoulos.

Βασίλειος Δρακόπουλος (Πανεπιστήμιο Θεσσαλίας)

Fractal interpolation functions with partial self-similarity

Natural geometric structures rarely exhibit exact self-similarity, which poses significant challenges for mathematical modelling. Existing approaches using fractal interpolation functions with partial self-similarity are typically based on Banach contraction mappings and therefore impose strong contraction conditions that restrict their descriptive power. In this work, a new framework for constructing generalised fractal interpolation functions with partial self-similarity

via Rakotch contraction mappings is introduced. By relaxing the contraction requirements, the proposed method provides additional flexibility and expressiveness in modelling intricate geometries. We describe the construction procedure and analyse the resulting interpolation functions. Several illustrative examples demonstrate that our approach can capture finer geometric features and more intricate natural patterns than traditional Banach-based fractal interpolation techniques.

Σπυρίδων Καμβύσης (Πανεπιστήμιο Κρήτης / FORTH)

Lax Pairs: Integrability and Chaos

Although the existence of a Lax pair is the first prerequisite for so-called "complete integrability", Lax pairs are definitely not sufficient. If one considers initial-boundary value problems for well-known "completely integrable" equations, integrability may or may not persist. We mention one example (Dirichlet problem for NLS) where it does and we provide a rigorous proof. We also mention numerical results by Arthur, Dorey, Parini for sine-Gordon with Robin condition, where even chaos can appear! A systematic rigorous study of this dichotomy is lacking.

This talk is based on a short invited paper with Dimitra Antonopoulou for an upcoming volume in memory of Peter Lax who passed away a few months ago.

Κωνσταντίνα Κυριακούδη (Πανεπιστήμιο Ιωαννίνων)

Mathematical Modelling of Blood Flow in Aneurysms

Mathematical modelling of real-life phenomena is an important tool, especially when it concerns health related problems. In this work, a mathematical model is presented for the description of blood flow under pathological conditions, such as in the case of an abdominal aortic aneurysm. To solve this problem, the Finite Volume Method is employed accompanied with the Generalized Curvilinear Coordinates system. Furthermore, the obtained non-linear algebraic system is solved with Newton's method, combined with the Trust Region method.

Joint work with Michail Xenos.

Άννα Λογιώτη (University of Stuttgart)

Global existence for long wave Hopf unstable spatially extended systems with a conservation law

We are interested in reaction-diffusion systems, with a conservation law, exhibiting a Hopf bifurcation at the spatial wave number k=0. With the help of a multiple scaling perturbation ansatz a Ginzburg-Landau equation coupled to a scalar conservation law can be derived as an amplitude system for the approximate description of the dynamics of the original reaction-diffusion system near the first instability. We use the amplitude system to show the global existence of all solutions starting in a small neighborhood of the weakly unstable ground state for original systems posed on a large spatial interval with periodic boundary conditions.

This is a joint work with N. Gauss, G. Schneider and D. Zimmermann.

Ανιέζα Μάλτσι (Weierstrass Institute for Applied Analysis and Stochastics, Berlin)

Modeling Thermoporoelasticity in Photoacoustics

Photoacoustic tomography (PAT) is a hybrid imaging modality that exploits the photoacoustic effect to achieve high-resolution visualization of biological tissue. In PAT, a short laser pulse irradiates the sample, where part of the optical energy is absorbed and converted into heat. The resulting thermoelastic expansion generates an initial pressure rise, which propagates as an ultrasonic wave and is subsequently measured by acoustic detectors. In this talk, we present a comprehensive modeling framework for photoacoustic imaging that incorporates thermal, porous, and elastic phenomena. The fully coupled system is derived using the GENERIC (General Equation for Non-Equilibrium Reversible-Irreversible Coupling) formalism, which provides a thermodynamically consistent foundation for linking the involved physical processes.

Μιχαήλ Ξένος (Πανεπιστήμιο Ιωαννίνων)

Mathematical Modelling of Blood Flow in Aneurysms

In nature, the vast majority of fluid flows are turbulent. Therefore, understanding turbulence and its temporal evolution is of great importance. In the context of dynamical systems, intermittency refers to the irregular alternation between phases of periodic behavior and chaotic dynamics, or between different forms of chaotic behavior.

Pomeau and Manneville identified three types of intermittency, in which a nearly periodic system exhibits irregularly spaced bursts of chaos. These types - Types I, II, and III - are associated with the approach to a saddle-node bifurcation, a subcritical Hopf bifurcation, and an inverse period-doubling bifurcation, respectively.

In this presentation, we begin with examples of dynamical systems of intermittency of all types, introducing simple mathematical models that capture aspects of intermittent behavior in turbulence. Furthermore, we study the intermittency phenomenon in the Burgers equation. An introduction to turbulence modeling, through the Boussinesq approximation and the two-equation turbulence model $k-\omega$ is presented. Additionally, numerical solutions are demonstrated for turbulent channel flow, utilizing the Unsteady Reynolds Averaged Navier-Stokes equations (URANS) and Large Eddy Simulation (LES) for the flow over the backward-facing step (BFS).

Joint work with Spyridon Katsoudas.

Ζωή Ράπτη (University of Illinois Urbana-Champaign)

Understanding the mechanisms driving epidemics through models and insufficient data

In six different lakes, during the same year and for the same host-pathogen system, epidemic outbreaks are different. We explore the drivers of the observed differences using a simple but realistic model and with the aid of limited field data. Our model is based on a system of nonlinear ODEs with time-varying coefficients and our observed data include the total host density and the density of early- and late-stage infectious hosts. We will present results from parameter identifiability analysis and model simulations. Several biologically relevant hypotheses explaining the observed patters will be investigated.

Αλεξάνδρα Τζέλλα (University of Birmingham / University of Edinburgh)

 $Boundary\ conditions\ for\ active\ Brownian\ particles$

We consider the transport of active particles inside a channel domain. These are treated using the Active Brownian Particle model (ABP), where particles move forward at constant speed but in a randomly-varying direction. We examine their accumulation near the boundary as the singular limit of a better-behaved model that includes small spatial diffusion. We present matched asymptotic approximations that describe their reduced dynamics at long times. In the reduced dynamics, the probability density for the particle position has a singular component localised on the boundary which is coupled in two-way fashion to to the smooth interior component. The results are illustrated by a number of numerical simulations.

Joint work with Daniel Loghin and Jacques Vanneste.