

Illuminating certain high-dimensional 1-unconditional convex bodies

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Abstract

Let us think of a convex body in \mathbb{R}^n (convex, compact set, with non-empty interior) as an opaque object, and let us place point light sources around it, wherever we want, to illuminate its entire surface. What is the minimum number of light sources that we need? The Hadwiger-Boltyanski illumination conjecture from 1960 states that we need at most as many light sources as for the n -dimensional hypercube, and more generally, as for n -dimensional parallelotopes. For the latter their illumination number is exactly 2^n , and they are conjectured to be the only equality cases.

The conjecture is still open in dimension 3 and above, and has only been fully settled for certain classes of convex bodies (e.g. zonoids, bodies of constant width, etc.). Moreover, there are some rare examples for which a basic, folklore argument could quickly lead to the upper bound 2^n , while at the same time understanding the equality cases has remained elusive for decades. One such example would be convex bodies very close to the cube, which was settled by Livshyts and Tikhomirov in 2017.

In this talk I will discuss another such instance, which comes from the class of 1-unconditional convex bodies, and which also “forces” us to settle the conjecture for a few more cases of 1-unconditional bodies. This is based on joint work with Wen Rui Sun, and our arguments are primarily combinatorial.