

A dichotomy of continuous finite element spaces and its application to energy-conservative Galerkin methods for nonlinear and dispersive wave equations

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Abstract

Recent progress in energy-conservative Galerkin semi-discretizations for nonlinear and dispersive wave equations has uncovered an unexpected pattern in convergence of the approximate solutions: when continuous Lagrange finite element spaces built from odd-degree polynomials are used, the method achieves the optimal convergence rate, whereas for even-degree polynomials the convergence rate is suboptimal. In this talk we show that this behavior has its origins in an inherent feature of the finite element spaces and is tied to the standard L^2 -projection of a function's derivative, which exhibits a super-approximation effect only in the odd-degree setting. We also illustrate how this characteristic determines the convergence of energy-conservative Galerkin methods for some nonlinear and dispersive wave equations.