An interplay between Bergman, Mahler, and Bourgain

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Abstract

We prove, in dimension 2, a sharp lower bound on the Bergman kernels of tube domains, verifying a conjecture of Blocki in this case. In the process, we discuss the connection between Bergman kernels of tube domains and Mahler volumes, building on the work of Nazarov, Blocki, and Berndtsson. This motivates the introduction of L_p analogues of Mahler volumes, which correspond to Bergman kernels when $p = 1$ and recover the classical Mahler volume for $p = \infty$. With these definitions, Blocki's conjecture is reformulated as an L_1-Mahler conjecture, which allows us to apply well-established techniques.

Time permitting, we will discuss the relationship between the Bergman metric and the isotropic constant, explaining how an old theorem of Kobayashi can be used to derive an upper bound on the isotropic constant.

This is partly joint work with Rubinstein and Berndtsson.