The infima of binary forms

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Abstract

For a binary form $P(x, y)$ of non-zero discriminant, what is the infimum of the values that P achieves, when (x, y) ranges over non-zero integer pairs? For such a form P, describing the set

$$\mathrm{Spec}(P) = \left\{ \inf_{(x,y)\in\mathbb{Z}^2\setminus\underline{0}} |P \circ \Lambda|(x, y), \text{ where } \Lambda \subset \mathbb{R}^2 \text{ is a lattice of determinant 1} \right\}$$

is a fundamental project in the geometry of numbers. For indefinite binary forms of degree 2, this question leads to the classical Markoff spectrum. For binary forms of degree higher than 2, not much is known for the distribution of these infima. In 1940, Mordell conjectured that for a binary cubic form $P(x, y)$, $\mathrm{Spec}(P)$ has a gap after its maximal value, a statement disproved later by Davenport, who constructed a sequence of infima converging to the top. As for $n \geq 4$, even less is known. In this talk, we discuss how to prove that for any binary form P, $Spec(P)$ is an interval of the form $[0, M_P]$, for some $M_P > 0$, giving a complete answer to the problem for all degrees n.