

Let $x, y, z > 0$ and $r := \sqrt{x^2 + y^2 + z^2}$. Show that

$$\begin{aligned} & xy \ln \left(\frac{r+z}{r-z} \right) + xz \ln \left(\frac{r+y}{r-y} \right) + yz \ln \left(\frac{r+x}{r-x} \right) \\ & > x^2 \arctan \left(\frac{yz}{xr} \right) + y^2 \arctan \left(\frac{xz}{yr} \right) + z^2 \arctan \left(\frac{xy}{zr} \right). \end{aligned}$$

As usual, for $t \in \mathbb{R}$ we have that $\arctan(t) \in (-\pi/2, \pi/2)$.